CHAPTER 5

ALGEBRAIC EXPRESSIONS

Index

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1.0 ALGEBRAIC EXPRESSIONS

1.1 Terms related to algebraic expression

Constant : A quantity which has a fixed value, i.e., whose value does not change is called a constant. Eg. 8, –7, 0, $6\frac{7}{8}$. etc. are all constants.

Variable: A symbol which can be assigned different numerical values is called a variable. In algebra, the variables are denoted by the letters of the English alphabet, viz.; a, b, c, ... x, y, z.

Eg. The perimeter P of a square of a side S is given by the formula, $P = 4 \times S$, Here 4 is a constant while P and S are variables.

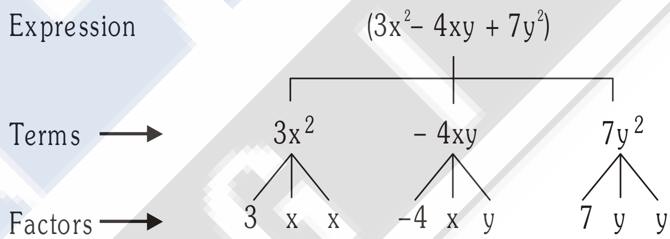
Algebraic expression: A combination of constants and variables connected by signs of fundamental operations (+, –, × and ÷) is called an algebraic expression.

Eg. $5 - 3x + 4x^2y$ is an algebraic expression consisting of three terms, namely 5, $-3x$ and $4x^2y$.

Factors: Each term of an algebraic expression consists of a product of constants and variables. A constant factor is called a numerical factor, while a variable factor is known as literal factor.

Eg. In the expression $3x^2 - 4xy + 7y^2$; $3x^2$, $-4xy$, $7y^2$ are all terms of the above expression. Each term is further made of factors, i.e., $3x^2$ can be written as $3 \times x \times x$, $-4xy$ as $-4 \times x \times y$ and $7y^2$ as $7 \times y \times y$. Here 3, x, x are factors of $3x^2$, –4, x, y are factors of –4xy and 7, y, y are factors of $7y^2$.

Thus an expression, its terms and factors of the terms can be represented by a tree diagram to make it easily comprehensible to you.



Coefficients: In a term, coefficient is either a numerical factor, an algebraic factor or the product of two or more factors.

Eg. In the algebraic expression 10 xy, 10 is the coefficient of xy, 10x is the coefficient of y and 10 y is the coefficient of x.

The numerical part is called the numerical coefficient and literal part or variable or variable part is called the literal coefficient.

When the numerical coefficient is not given. It is always understood to be 1.

Eg. Term	Numerical coefficient	Literal coefficient
$84x^2z$	84	x^2z
–32xy	–32	xy

Like terms: Like terms or similar terms are terms which have the same literal factor. They may differ in their numerical coefficient.

Eg. (i) $9x^2$, $5x^2$ and $-2x^2$ are like terms. (ii) $-6x^2y$ and $4yx^2$ are like terms.

Unlike terms: Unlike terms are those which have different literal factors.

Eg. $7x^2$ and $9y^2$, the literal factors are x^2 in $7x^2$ and y^2 in $7y^2$, which are different.

Illustrations

Illustration 1. Draw a tree diagram of the expression $4x^2 + 5y - 8z + 1$.

Solution

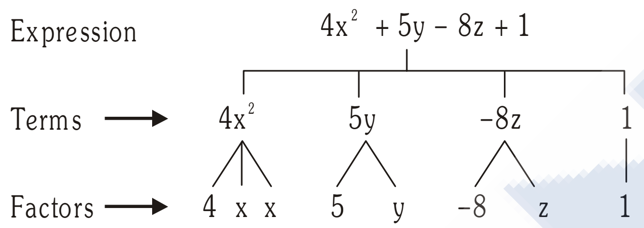


Illustration 2. (i) Write down the numerical as well as the literal coefficient of each of the following expression.

(a) $7x^2y^2z$ (b) $-28x^2yzq^2$

(ii) Identify the like terms in each of the following :

(a) $9x^2, 8xy, -2xy, \frac{5}{2}xy$ (b) $8m^2n^2, -mp^2, -m^2n^2, nm^2$

(iii) Find the coefficient of x in the following :

(a) x (b) $-3xy^2$ (c) $4p^2x$

Solution (i)

Expression	numerical coefficient	literal coefficient
(a) $7x^2y^2z$	7	x^2y^2z
(b) $-28x^2yzq^2$	–28	x^2yzq^2

(ii) (a) Like terms: $8xy, -2xy, \frac{5}{2}xy$

(b) Like terms: $8m^2n^2, -m^2n^2$

(iii) (a) 1 (b) $-3y^2$ (c) $4p^2$

1.2 Various types of algebraic expressions

Monomials: An algebraic expression which contains only one term, is called a monomial.

Thus $5x, 2xy, -3a^2b, -7$, etc are all monomials.

Binomials: An algebraic expression containing two terms is called a binomial.

Thus, $(2a + 3b), (8 - 3x), (x^2 - 4xy^2)$ etc. are all binomials.

Trinomials: An algebraic expression containing three terms is called a trinomial.

Thus $(a + 2b + 5c), (x + 2y - 3z), (x^3 - y^3 - z^3)$, etc, are all trinomials.

Quadrinomials: An algebraic expression containing four terms is called a quadrinomials.

Thus $(x + y + z - 5), (x^3 + y^3 + z^3 + 3xyz)$, etc, are all quadrinomials.

Polynomials: An expression containing two or more terms is called a polynomial.

An algebraic expression of the form $a + bx + cx^2 + dx^3 + ...$, where a, b, c, d are constants and x is a variable is called a polynomial in x.

The degree of the polynomial is the greatest power of the variable present in the polynomial.

Eg. $9x^4 - \frac{9}{2}x^2 - \frac{7}{5}x - 2$ is a polynomial in x of degree 4.

Illustrations

Illustration 3. (i) Which of the following is a polynomial?

(a) $7a^2 - 3ab + 5b^2 + 9$ (b) $\frac{9}{b^2} + a + ab$

(ii) State the degree of the polynomial: $a^2b^2 - ab + 3ab^2$.

Solution. (i) (a) $7a^2 - 3ab + 5b^2 + 9$ is a polynomial in a and b. The sum of the powers of variable of each term is 2, 2, 2 respectively.

(b) $\frac{9}{b^2} + a + ab$ is not a polynomial in a and b.

The sum of the powers of variables of each term is –2, 1, 2 respectively.

(ii) $a^2b^2 - ab + 3ab^2$ is a polynomial in a and b.

The sum of the powers of variables of each term is 4, 2, 3 respectively.

$\therefore$ The degree is highest power of variable in the expression. So it is a 4 degree polynomial.

1.3 Operations on algebraic expressions

Addition of algebraic expressions

While adding algebraic expressions, we collect the like terms and add them. The sum of several like terms is another like term whose coefficient is the sum of the coefficients of those like terms.

Subtraction of algebraic expressions

The difference of two like terms is a like term whose coefficient is the difference of the numerical coefficients of the two like terms.

Rule for subtraction of algebraic expressions:

Change the sign of each term of the expression to be subtracted and then add.

Multiplication of algebraic expressions

Before taking up the product of algebraic expression, let us look at two simple rules.

(i) The product of two factors with like signs is positive, and the product of two factors with unlike signs is negative.

(ii) If a is any variable and m, n are positive integers then $a^m \times a^n = (a^{m+n})$

Thus, $x^3 . x^5 = x^{3+5} = x^8$ and $x^{7+1} = x^8$, etc.

Multiplication of monomials

Rules:

(i) The coefficient of the product of two monomials is equal to the product of their coefficients.

(ii) The variable part in the product of two monomials is equal to the product of the variables in the given monomials.

These rules may be extended for the product of three or more monomials.

Multiplication of a monomial and a binomial

Let p, q and r be three monomials.

Then, by distributive law of multiplication over addition, we have

$$p \times (q + r) = (p \times q) + (p \times r)$$

Multiplication of two Binomials

Suppose (a + b) and (c + d) are two binomials. By using the distributive law of multiplication over addition twice, we may find their product as given below:

$$\begin{aligned}(a + b) \times (c + d) &= a \times (c + d) + b \times (c + d)\\ &= (a \times c + a \times d) + (b \times c + b \times d)\\ &= ac + ad + bc + bd.\end{aligned}$$

This method is known as the horizontal method.

Division

Division of Monomials

To divide one algebraic term by another, the power or exponent rule for division is used.

(i) The power of the all factors in the denominator is subtracted from the like factors in the numerator.

(ii) The numerical coefficient of the numerator and denominator is divided by the LCM.

If either, or both terms are fractions, then the division sign is changed to multiplication and the second term (fraction) is inverted.

Then division is then carried out as explained in (1) above or simplify by reducing to the lowest terms.

Division of Polynomials

(i) To divide a polynomial by a monomial, divide each term of the polynomial by the monomial and then simplify.

(ii) To divide a polynomial by a polynomial, follow the steps described below:

(a) Arrange the dividend and divisor in descending or ascending order of power of any one particular variable. (The same variable must be taken in both cases).

(b) Divide the first term of the dividend by the first term of the divisor and obtain the quotient.

(c) Divisor × quotient = dividend. Hence multiply the quotient by each term of the divisor and write each product below the like term of the dividend.

(d) Change the sign and subtract.

(e) Repeat steps b, c, d with the next divisor.

Illustrations

Illustration 4. Add: $5x^2 - 7x + 3$, $-8x^2 + 2x - 5$ and $7x^2 - x - 2$.

Solution Required sum

$= (5x^2 - 7x + 3) + (-8x^2 + 2x - 5) + (7x^2 - x - 2)$

$= 5x^2 - 8x^2 + 7x^2 - 7x + 2x - x + 3 - 5 - 2$ [collecting like terms]

$= (5 - 8 + 7)x^2 + (-7 + 2 - 1)x + (3 - 5 - 2)$ [adding like terms]

$= 4x^2 - 6x - 4$.

Illustration 5. Subtract $(2x^2 - 5x + 7)$ from $(3x^2 + 4x - 6)$.

Solution We have:

$(3x^2 + 4x - 6) - (2x^2 - 5x + 7)$

$= 3x^2 + 4x - 6 - 2x^2 + 5x - 7$

$= (3 - 2)x^2 + (4 + 5)x + (-6 - 7)$

$= x^2 + 9x - 13$.

Illustration 6. Multiply:

(i) $-8ab^2c$, $3a^2b$ and $-\frac{1}{6}$ (ii) $\frac{5}{8}a^3b^2$, $12a^2b$ and $6c$

(iii) 4, $\frac{5}{12}x$ and $-8x^2y$ (iv) $-a$, a^2bc and $\frac{-2}{5}ab^2c^2$

Solution (i) $(-8ab^2c) \times (3a^2b) \times \left(-\frac{1}{6}\right)$

$$= \left(-8 \times 3 \times \frac{-1}{6}\right) \times (a \times a^2 \times b^2 \times b \times c) = 4a^{(1+2)} \times b^{(2+1)} \times c = 4a^3b^3c.$$

(ii) $\left(\frac{5}{8}a^3b^2\right) \times (12a^2b) \times (6c)$

$$= \left(\frac{5}{8} \times 12 \times 6\right) \times (a^3 \times a^2 \times b^2 \times b \times c) = 45 \times a^{(3+2)} \times b^{(2+1)} \times c = 45a^5b^3c.$$

(iii) $4 \times \left(\frac{5}{12}x\right) \times (-8x^2y)$

$$= \left[4 \times \frac{5}{12} \times (-8)\right] \times (x \times x^2 \times y) = \frac{-40}{3} \times x^{(1+2)} \times y = \frac{-40}{3}x^3y$$

(iv) $(-a) \times (a^2bc) \times \left(\frac{-2}{5}ab^2c^2\right) = \left(-1 \times \frac{-2}{5}\right) \times a \times a^2 \times a \times b \times b^2 \times c \times c^2$

$$= \frac{2}{5} \times a^{(1+2+1)} \times b^{(1+2)} \times c^{(1+2)} = \frac{2}{5}a^4b^3c^3.$$

Illustration 7. Multiply: $\frac{9}{2}x^2y$ by$(x + 2y)$.

Solution $\frac{9}{2}x^2y \times (x + 2y) = \left(\frac{9}{2}x^2y \times x\right) + \left(\frac{9}{2}x^2y \times 2y\right)$ [by distributive law]

$$= \left(\frac{9}{2} \times x^2 \times x \times y\right) + \left(\frac{9}{2} \times 2 \times x^2 \times y \times y\right)$$

$$= \left\{\frac{9}{2}x^{(2+1)} \times y\right\} + \left\{9 \times x^2 \times y^{(1+1)}\right\} = \frac{9}{2}x^3y + 9x^2y^2.$$

Illustration 8. Multiply : $\left(\frac{1}{5}x - \frac{1}{4}y\right)$ and $(5x^2 - 4y^2)$.

Solution Using horizontal method, we have

$$\left(\frac{1}{5}x - \frac{1}{4}y\right) \times (5x^2 - 4y^2) = \frac{1}{5}x(5x^2 - 4y^2) - \frac{1}{4}y(5x^2 - 4y^2)$$

$$= \left(\frac{1}{5}x \times 5x^2\right) - \left(\frac{1}{5}x \times 4y^2\right) - \left(\frac{1}{4}y \times 5x^2\right) + \left(\frac{1}{4}y \times 4y^2\right)$$

$$= x^3 - \frac{4}{5}xy^2 - \frac{5}{4}x^2y + y^3.$$

Column method

$$\begin{array}{r}
\frac{1}{5}x - \frac{1}{4}y \\
\times(5x^2 - 4y^2) \\
\hline
x^3 - \frac{5}{4}x^2y \\
-\frac{4}{5}xy^2 + y^3 \\
\hline
x^3 - \frac{5}{4}x^2y - \frac{4}{5}xy^2 + y^3 \\
\hline
\end{array}$$

Illustration 9. (i) $4a \div a$ (ii) $4a^2b \div ab$ (iii) $9a^4b^2 \div 3a^2b$

Solution (i) $4a \div a = \frac{4a}{a} = 4a^{1-1} = 4a^0 = 4$

(ii) $4a^2b \div ab = \frac{4a^2b}{ab} = 4a^{2-1}b^{1-1} = 4a$

(iii) $9a^4b^2 \div 3a^2b = \frac{9a^4b^2}{3a^2b} = 3a^{4-2}b^{2-1} = 3a^2b = 3a^2b$

Illustration 10. $(6a^2 + a) \div 4a$

Solution $\frac{6a^2}{4a} + \frac{a}{4a} = \frac{3a}{2} + \frac{1}{4} = \frac{6a+1}{4}$

1.4 Value of an expression

The value of an expression can be found by substituting the given value of literal in the expression.
Eg. Value of $x^2 + 2x + 1$ when $x = 5$ is $5^2 + 2 \times 5 + 1 = 25 + 10 + 1 = 36$.

Illustrations

Illustration 11. If $p = -2$, find the value of:
(i) $4p + 8$ (ii) $-3p^2 + 5p + 7$ (iii) $-2p^3 - 3p^2 + 4p + 7$

Solution (i) Value of $(4p + 8)$, when $p = -2$ is $4(-2) + 8 = -8 + 8 = 0$
(ii) Value of $(-3p^2 + 5p + 7)$ when $p = -2$ is $-3(-2)^2 + 5(-2) + 7$
$= -3 \times 4 - 10 + 7 = -12 - 10 + 7 = -15$
(iii) Value of $(-2p^3 - 3p^2 + 4p + 7)$
when $p = -2$ is $-2(-2)^3 - 3(-2)^2 + 4(-2) + 7$
$= 16 - 12 - 8 + 7 = 3$

Illustration 12. Find the value of a that will make the expression $3x^2 - 8x + a$ equal to 5, at $x = 2$.

Solution Value of $3x^2 - 8x + a$, when $x = 2$ is $3(2)^2 - 8(2) + a = 12 - 16 + a = a - 4$
Now, $a - 4 = 5 \Rightarrow a = 9$

1.5 Formulas and rules using algebraic expressions

Perimeter formulae

(i) Perimeter of an equilateral triangle $= 3\ell$, where ℓ is the length of the side of the equilateral triangle.
(ii) Perimeter of a square $= 4\ell$, where ℓ is the length of the side of the square.
(iii) Perimeter of a regular pentagon $= 5\ell$, where ℓ is the length of the side of the pentagon, etc.

Area formulae

(i) Area of a square $= \ell^2$, where ℓ is the side of the square.
(ii) Area of a rectangle $= \ell \times b$, where ℓ and b are respectively the length and the breadth of the rectangle.
(iii) Area of triangle $= \dfrac{b \times h}{2}$, where b is the base of the triangle and h is the height of the triangle.

CHECK POST-1

1. Draw a tree diagram for each of the following expression:
(i) $5a^2 - 17ab$ (ii) $9xy + 7y$ (iii) $2x^2y - 6xy + 8$

2. What should be added to $x^2 + xy + y^2$ to obtain $3x^2 + 4xy$?

3. What should be subtracted from $2a + 8b + 10$ to get $-4a + 3b + 12$?

4. What should be value of p if the value of $3x^2 + 4x - p$ is 5, when
(i) $x = -1$ (ii) $x = 1$

5. Find the value of the expression
$a^3 - b^3 + 3ab(a - b)$ when $a = -3$, $b = 1$

GOLDEN KEY POINTS

- Algebra is like arithmetic but uses letters and symbols to stand for numbers. Unknown numbers in algebra are represented by letters such as 'x' and 'y' and they are related to form equations.
- Every polynomial is an expression but every expression need not be a polynomial. An expression containing a term of the form $\frac{1}{x}, \frac{1}{x^2}, \frac{x}{y}, \frac{x^2}{y}$ etc. is not a polynomial.
- At the time of adding and subtracting the expression arrange the expressions so that the like terms are grouped together or arrange the expression in rows so that like terms appear in columns and then add or subtract.
- Unlike terms can not be added and also can not be subtracted.
- If the polynomial is in two or more variables then the sum of the powers of the variables in each terms is taken and the greatest sum is the degree of the polynomial.
- The number for which the value of a polynomial is zero, is called zero of the polynomial.
- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)(a - b) = a^2 - b^2$

SOME WORKED OUT ILLUSTRATIONS

Illustration 1.

Add: $\left(3x^2-\frac{1}{5}x+\frac{7}{3}\right)+\left(-\frac{1}{4}x^2+\frac{1}{3}x-\frac{1}{6}\right)+\left(-2x^2-\frac{1}{2}x+5\right)$

Solution

Required sum

$$=\left(3x^2-\frac{1}{5}x+\frac{7}{3}\right)+\left(-\frac{1}{4}x^2+\frac{1}{3}x-\frac{1}{6}\right)+\left(-2x^2-\frac{1}{2}x+5\right)$$

$$=3x^2-\frac{1}{4}x^2-2x^2-\frac{1}{5}x+\frac{1}{3}x-\frac{1}{2}x+\frac{7}{3}-\frac{1}{6}+5 \text{ [collecting like terms]}$$

$$=\left(3-\frac{1}{4}-2\right)x^2+\left(-\frac{1}{5}+\frac{1}{3}-\frac{1}{2}\right)x+\left(\frac{7}{3}-\frac{1}{6}+5\right) \quad \text{[adding like terms]}$$

$$=\left(\frac{12-1-8}{4}\right)x^2+\left(\frac{-6+10-15}{30}\right)x+\left(\frac{14-1+30}{6}\right)$$

$$=\frac{3}{4}x^2-\frac{11}{30}x+\frac{43}{6}.$$

Illustration 2.

Take away $\left(\frac{8}{5}x^2-\frac{2}{3}x^3+\frac{3}{2}x-1\right)$ from $\left(\frac{x^3}{5}-\frac{3}{2}x^2+\frac{2}{3}x+\frac{1}{4}\right)$.

Solution

We have:

$$\left(\frac{x^3}{5}-\frac{3}{2}x^2+\frac{2}{3}x+\frac{1}{4}\right)-\left(\frac{8}{5}x^2-\frac{2}{3}x^3+\frac{3}{2}x-1\right)$$

$$=\frac{x^3}{5}-\frac{3}{2}x^2+\frac{2}{3}x+\frac{1}{4}-\frac{8}{5}x^2+\frac{2}{3}x^3-\frac{3}{2}x+1$$

$$=\left(\frac{1}{5}+\frac{2}{3}\right)x^3+\left(-\frac{3}{2}-\frac{8}{5}\right)x^2+\left(\frac{2}{3}-\frac{3}{2}\right)x+\left(\frac{1}{4}+1\right)$$

$$=\frac{(3+10)}{15}x^3+\left(\frac{-15-16}{10}\right)x^2+\frac{(4-9)}{6}x+\frac{(1+4)}{4}$$

$$=\frac{13}{15}x^3-\frac{31}{10}x^2-\frac{5}{6}x+\frac{5}{4}.$$

JPR\Comp.251\Allen-Junior wing (2020-21)\maths\VII\Unit-1\05_Algebraic Expression

Illustration 3.

Multiply: $(-5x^2y)$, $\left(\frac{-2}{3}xy^2z\right)$, $\left(\frac{8}{15}xyz^2\right)$ and $\left(\frac{-1}{4}z\right)$ Verify the result when x = 1, y = 2 and z = 3.

Solution

We have $(-5x^2y) \times \left(\frac{-2}{3}xy^2z\right) \times \left(\frac{8}{15}xyz^2\right) \times \left(\frac{-1}{4}z\right)$

$$=\left(-5 \times \frac{-2}{3} \times \frac{8}{15} \times \frac{-1}{4}\right) \times (x^2 \times x \times x \times y \times y^2 \times y \times z \times z^2 \times z)$$

$$=\frac{-4}{9} \times x^{(2+1+1)} \times y^{(1+2+1)} \times z^{(1+2+1)} = \frac{-4}{9}x^4y^4z^4.$$

Verify: $(-5x^2y) = -5 \times (1)^2 \times (2) = -10$

$$\left(\frac{-2}{3}xy^2z\right) = \frac{-2}{3} \times (1) \times (2)^2 \times 3 = -8$$

$$\left(\frac{8}{15}xyz^2\right) = \frac{8}{15} \times 1 \times 2 \times 3^2 = \frac{8}{\cancel{15}_5} \times 2 \times \cancel{9}^3 = \frac{48}{5}$$

$$\left(-\frac{1}{4}z\right) = -\frac{1}{4} \times 3 = \frac{-3}{4} \quad ; \quad (-5x^2y) \times \left(\frac{-2}{3}xy^2z\right) \times \left(\frac{8}{15}xyz^2\right) \times \left(-\frac{1}{4}z\right)$$

$$(-10) \times (-8) \times \left(-\frac{3}{4}\right) \times \left(\frac{48}{5}\right) = -576$$

$$\frac{-4}{9}x^4y^4z^4 = \frac{-4}{9}(1)^4 \times (2)^4 \times (3)^4 = \frac{-4}{\cancel{9}_1} \times 16 \times \cancel{81}^9 = -576$$

Hence $(-5x^2y) \times \left(\frac{-2}{3}xy^2z\right) \times \left(\frac{8}{15}xyz^2\right) \times \left(-\frac{1}{4}z\right) = \left(-\frac{4}{9}\right)x^4y^4z^4$

Illustration 4.

Multiply: $\left(3x - \frac{4}{5}y^2x\right)$ by $\frac{1}{2}xy$.

Solution

We have

$$\left(3x - \frac{4}{5}y^2x\right) \times \frac{1}{2}xy = \left(3x \times \frac{1}{2}xy\right) - \left(\frac{4}{5}y^2x \times \frac{1}{2}xy\right)$$

$$=\left(3 \times \frac{1}{2} \times x \times x \times y\right) - \left(\frac{4}{5} \times \frac{1}{2} \times y^2 \times y \times x \times x\right)$$

$$=\left\{\frac{3}{2} \times x^{(1+1)} \times y\right\} - \left\{\frac{2}{5} \times y^{(2+1)} \times x^{(1+1)}\right\} = \left(\frac{3}{2}x^2y\right) - \left(\frac{2}{5}y^3x^2\right).$$

column method we have

$$\begin{array}{c} 3x - \frac{4}{5}y^2x \\ \frac{1}{2}xy \\ \hline \frac{3}{2}x^2y - \frac{2}{5}y^3x^2 \\ \hline \end{array}$$

Illustration 5.

Multiply: $(5x^2 - 6x + 9)$ by $(2x - 3)$.

Solution

By column method, we have

$$\begin{array}{rll} & 5x^2 - 6x + 9 & \\ & \times 2x - 3 & \\ \hline & 10x^3 - 12x^2 + 18x & \text{multiplication by } 2x \\ & -15x^2 + 18x - 27 & \text{multiplication by } -3 \\ \hline \text{Add:} & \underline{10x^3 - 27x^2 + 36x - 27} & \text{multiplication by } (2x-3) \end{array}$$

$\therefore$ $(5x^2 - 6x + 9) \times (2x - 3) = 10x^3 - 27x^2 + 36x - 27.$

Illustration 6.

$$\frac{9a^8b^4}{3a^4b^2} \div \frac{3a^2b^2}{6a}$$

Solution

$$\frac{9a^8b^4}{3a^4b^2} \times \frac{6a}{3a^2b^2}$$

$= 6a^{8+1-4-2}b^{4-2-2}$

$= 6a^{9-6}b^{4-4}$

$= 6a^3b^0 = 6a^3$

Illustration 7.

If a = 1, b = –1, find the value of:

(i) $a^2 + b^2$ (ii) $a^2 + ab + b^2$ (iii) $a^2 - b^2$

Solution

(i) Value of $a^2 + b^2$, when a = 1, b = –1 is $(1)^2 + (-1)^2 = 1 + 1 = 2$

(ii) Value of $a^2 + ab + b^2$, when a = 1, b = –1 is

$(1)^2 + (1)(-1) + (-1)^2 = 1 - 1 + 1 = 1$

(iii) Value of $a^2 - b^2$, when a = 1, b = –1 is $(1)^2 - (-1)^2 = 1 - 1 = 0$

Illustration 8.

Find the value of m if the expression $x^4 - 5x^3 + mx - 4$ equals –1 when x = –1.

Solution

The value of $(x^4 - 5x^3 + mx - 4)$ is –1, when x = –1

$\therefore$ $(-1)^4 - 5(-1)^3 + m(-1) - 4 = -1$

$\Rightarrow$ $1 + 5 - m - 4 = -1$

$\Rightarrow$ $2 - m = -1 \Rightarrow -m = -1 - 2 = -3$

$\Rightarrow$ $m = 3$

EXERCISE – 1 ELEMENTARY

1. Which of the following expressions is not a polynomial?

(A) $6y^3 + 5y^2 - 2y - 9$ (B) $-\frac{2}{9}x^2y + \frac{4}{13}x^2y^2 + 6y^3$

(C) $(a^3 - 8a)(x^4 + 6)$ (D) $\frac{5x^4 + 7x^2y^2 - 8}{y}$

2. Which of the following expression is a polynomial?

(A) $y^2 + \sqrt{2}y(x-4) + x$ (B) $\sqrt[3]{9x} + x^4 - x$ (C) $a^{-\frac{1}{2}} + \sqrt{5}a + 6$ (D) $4\sqrt{x} + xy - 1$

3. What is the degree of the polynomial $2a^2 + 4b^8$?
(A) 2 (B) 10 (C) 8 (D) 0

4. Degree of a constant term is
(A) 1 (B) 0 (C) 2 (D) not defined

5. Degree of the polynomial $(a^2 + 1)(a + 2)(a^3 + 3)$ is
(A) 3 (B) 6 (C) 2 (D) 7

6. What is the degree of –127 ?
(A) 1 (B) 0 (C) 2 (D) Not defined

7. Write like terms from $2x^2$, $2x$, $2x^3$, $-5x$, $11x^4$
(A) $2x^2$, $2x$ (B) $2x$, $-5x$ (C) $2x^3$, $11x^4$ (D) $2x^2$, $2x^3$

8. $4x^2 + 2x + 4$ is a
(A) Monomial (B) Binomial (C) Trinomial (D) None of these

9. On adding $(1 - z)$ to $(z - 1)$, we get
(A) $2z$ (B) $2z + 2$ (C) 0 (D) $2z - 2$

10. How much is $(-a - 2b - 3c)$ less than $(a + 3b - 5c)$?
(A) $-2a - 5b + 2c$ (B) $2a + 5b - 2c$ (C) $2a + 6b - 15c$ (D) $2a - 5b + 15c$

11. The sum of three expressions is $x^2 + y^2 + z^2$. If two of them are $4x^2 - 5y^2 + 3z^2$ and $-3x^2 + 4y^2 - 2z^2$, the third expression is
(A) $2x^2 + 2z^2$ (B) $2y^2$ (C) $2x^2 + 2y^2 - z^2$ (D) $2y^2 + 2z^2$

12. If $P = 3x - 4y - 8z$, $Q = -10y + 7x + 11z$ and $R = 19z - 6y + 4x$, then $P - Q + R$ is equal to
(A) $13x - 20y + 16z$ (B) 0 (C) $x + y + z$ (D) $2x - 4y + 3z$

13. Sum of $(3a - 7b)$, $(5a + 8b)$ and $(-6a + 9b)$ is
(A) $2a + 10b$ (B) $2a + 5b$ (C) 2 (D) 0

14. The product of $4a^2$, $-6b^2$ and $3a^2b^2$ is
(A) a^2b^2 (B) $13a^4b^4$ (C) $-72a^4b^4$ (D) a^4b^4

15. $(14x^2yz - 28x^2y^2z^3 + 32y^2z^2) \div (-4xy)$ is equal to
(A) $\frac{7}{2}yz + 7xyz^2 + 8xyz$ (B) $-\frac{7}{2}xz + 7xyz^3 - \frac{8yz^2}{x}$
(C) $-\frac{7}{2}xz - 7xyz^3 + \frac{8yz^2}{x}$ (D) $\frac{7}{2}xz - 7xyz^2 - \frac{8yz^2}{x}$

16. The product of $\left(\frac{1}{5}x^2 - \frac{1}{6}y^2\right)(5x^2 + 6y^2)$ is
(A) 1 (B) $x^4 + \frac{11}{60}x^2y^2 + y^4$ (C) $x^4 + \frac{11}{30}x^2y^2 - y^4$ (D) $x^4 - \frac{11}{30}x^2y^2 - y^4$

17. The product $(x + 2)(x^2 - 2x + 4)$ is equal to
(A) $x^3 + 8$ (B) $x^3 - 8$ (C) $x^3 - 4x^2 + 4x - 8$ (D) $x^3 + 4x^2 + 2x + 8$

18. The value of the expression $a^3 - 3a^2 + 3a - 1$ for $a = -1$ is
(A) –8 (B) –2 (C) 0 (D) 2

19. The value of the polynomial $p(x) = \ell x + m$ at $x = -\frac{m}{\ell}$ is
(A) 0 (B) 1 (C) 3 (D) 4

20. The value of the polynomial $p(x) = \frac{-11}{9} - \frac{17}{26}x$ at $x = \frac{13}{9}$ is
(A) $\frac{-10}{7}$ (B) $\frac{-13}{6}$ (C) $\frac{-15}{7}$ (D) $\frac{-17}{6}$

21. The value of the polynomial $p(x) = 4 + x^3 - x + 2x^2$ at $x = -2$ is
(A) 2 (B) 4 (C) 6 (D) 8

22. What must be added to $x^3 + 3x - 8$ to get $3x^3 + x^2 + x$?
(A) $2x^3 + x^2 - 3x + 14$ (B) $2x^3 + x^2 - 6x - 14$
(C) $2x^3 + x^2 - 2x + 8$ (D) $2x^3 + x^2 - 14$

23. The value of $5p^2 - 4p(-p + q)$ is
(A) $p^2 + 4pq$ (B) $9p^2 + 4pq$ (C) $9p^2 - 4pq$ (D) $p^2 - 4pq$

24. If $X = 2x^2$, $Y = 4x^6 - 6x^4$, then find the value of X/Y when $x = 1$.
(A) –4 (B) 1 (C) –1 (D) 3

25. $x^2 + 2x + 3x^2 + 2 + 4x + 7 =$ __________
(A) $4x^2 + 6x + 11$ (B) $6x^2 + 6x + 6$ (C) $4x^2 + 6x + 9$ (D) $4x^2 + 6x + 12$

EXERCISE – 2 SEASONED

1. $(x + 4)(x + 3) - (x - 4)(x - 3)$ is equal to
(A) $2x^2 - 14x + 24$ (B) $2x^2 + 14x - 24$ (C) $14x$ (D) 24

2. Find the value of the expression $z^3 - 2(z - 10)$ for $z = 10$.
(A) 10 (B) 100 (C) 1000 (D) 10000

3. Find value of the expression $-x + 2$ for $x = -2$.
(A) 1 (B) 2 (C) 3 (D) 4

4. Find the value of the expression $3p + 7$ for $p = -2$.
(A) 1 (B) –1 (C) 2 (D) –2

5. Find the value of the expression $a^3 + b^3 + c^3 - 3abc$ for $a = 2, b = 3, c = 4$.
(A) 3 (B) 6 (C) 9 (D) 27

6. If the value of the expression $x^2 - 5x + k$ for $x = 0$ is 5, then the value of k is
(A) 2 (B) 3 (C) 4 (D) 5

7. $x + y - (z - x - [y + z - (x + y - \{z + x - (y + z + x)\})])$ is equal to
(A) 3x (B) 2y (C) x (D) 0

8. The remainder when $x^3 - 2x^2 + 4x$ is divided by x^2 is
(A) 1 (B) $x-2+\frac{4}{x}$ (C) 4x (D) $4x - 2x^2$

9. What should be added to $\frac{1}{x}$, to make it equal to x?
(A) $\frac{x^2-x}{x^2}$ (B) $\frac{x}{x^2-1}$ (C) $\frac{x^2+1}{x}$ (D) $\frac{x^2-1}{x}$

10. $\frac{3}{4}(a+y)-\left[y+a-\frac{1}{3}\left(y+a-\frac{1}{4}(a+y)\right)\right]$ is equal :
(A) a + y (B) 3a (C) –4y (D) 0

11. Find the value of the expression $a^2 + ab + 1$ for $a = 0, b = 1$.
(A) 0 (B) 1 (C) –1 (D) 2

12. Find the value of the expression $3x + 5(x - 2)$ for $x = 2$.
(A) 2 (B) 4 (C) 5 (D) 6

13. Simplify $3x(4x - 5) + 3$ for $x = 3$.
(A) 60 (B) 66 (C) 63 (D) 69

14. If $4x + 3y - 2a = 2$ for $x = -2$ and $y = 2$, then find the value of a.
(A) –1 (B) 2 (C) –2 (D) 1

15. The sides of a rectangle are $2a + 7c$ and $5a - 9c$. Find its perimeter.
(A) $14a - 4c$ (B) $14a + 4c$ (C) $7a + 2c$ (D) $7a - 2c$

EXERCISE – 3 SUBJECTIVE

Very short answer type questions

1. Group the like term together

(i) $6a, 5b, -7a, -3b$

(ii) $11x, -3y, 6y, y, 8x, -6x$ and $5x$

(iii) $6m, 8mn, -16mn, -10m, -8m^2, 15m^2$ and $14mn$

(iv) $14x^2y, 2xy^2, 6xy, -6yx, 5yx^2$ and $14y^2x$

2. Identify the type of algebraic expression and state the degree of each

(i) $3 - 8x$

(ii) $x^2 + 4y^2$

(iii) $8x^2y^6 + x^5$

(iv) $-6a^3 + 5a^3b - ab^3 + 9b^4$

(v) $6a^3 - 2a^2b^2 + 18a^4 - 16a^3b^4 - 22a^6 + 16b^8$

(vi) $18a^4b^3 - 6a^3b^6 + 20a^7 - 16b^8 + 20a^4b^4$

3. Add the following: (Use the row method)

(i) $-3p + 6q, 3p - 3q$

(ii) $8p - 6q, 3p - 4q, 12q - 12p$

(iii) $6x - 4y - 2z, 3y - 2x + z, 4z - x - 2y$

(iv) $15p^2 + 10pq - 6q^2, -8pq + 10p^2 - 9q^2$

4. Subtract the first expression from the second:

(i) $16a, 18a$

(ii) $-12x, 10$

(iii) $-4bc, 12$

5. Write down the numerical as well as the literal coefficient of each of the following monomials:

(i) $-10x^2y$

(ii) $\frac{6}{13}a^2bc$

(iii) $-\frac{5}{9}abc$

(iv) $-pq$

(v) $\frac{-5xy}{9z}$

(vi) $-8x^2y^2z^2$

6. Identify polynomials from the given algebraic expressions:

(i) $\frac{4}{9}x^2 - \frac{4}{9}x^2 + 4$

(ii) $\frac{2}{x} + \frac{3}{x^2} - 4x^2 + 3x + 2$

(iii) $3a^3 + 4a^2b + 8b^3$

(iv) $x^2 - \frac{1}{x^2} + 2$

Short answer type questions

7. Add the following: (Use the row method)

(i) $5a^2 + 8a - 4, 3a^2 - 4a + 11, -4a^2 - 3a + 6$

(ii) $p^2 + q^2 + 2pq, 3p^2 + 3r^2 + 6pr, 9r^2 - 4p^2 + 6pr$

(iii) $a^2b + ab^2 - abc - b^2c, 5a^2b - 6ab^2 - 3abc - 2cb^2$

(iv) $\frac{2}{5}x - \frac{3}{2}y + 2z, \frac{x}{3} + \frac{y}{4} - \frac{z}{2}$ and $\frac{-4x}{5} + \frac{y}{3} - \frac{3z}{2}$

(v) $\frac{5x^2}{3} - \frac{7x}{2} + 11, \frac{7x^2}{2} + \frac{5x}{3} - 6$ and $\frac{7x^2}{3} - \frac{4x}{3} - 2$

(vi) $2 - \frac{2}{5}x^3 + 3x^2, x^3 - \frac{x^2}{4} + 5$ and $\frac{2x^3}{5} + \frac{3x^2}{4} - 10$

8. Subtract the first expression from the second:

(i) $8p^2 - 6p^2q + 8q^2$, $12p^2 - 16p^2q + 8pq^2 + 9q^2$

(ii) $9p^3 + 4p^2 + 6p - 4$, $12p^3 - 12$

(iii) $6a^4 - 2a^2b^2 + 2b^4$, $8a^4 - 6a^2b^2 + 8a^3b - 8a^2b - b^4$

(iv) $\frac{6x^2}{5} - \frac{2x}{3} + 10$, $\frac{8x^2}{3} - \frac{3x}{2} + 8$

(v) $\frac{5x^2}{2} - \frac{2y^2}{3} + \frac{xy}{6}$, $\frac{3x^2}{2} - \frac{3y^2}{4} + \frac{xy}{6}$

9. (i) Subtract $9a^2 + 19b + 21$ from $12a^2 + 26b + 32$

(ii) Add $8p + 3q + 9$ to the difference of $16p + 5q + 10$ and $6p - 3q - 1$

(iii) Subtract the sum of $2a + 3b + 7$ and $9a - 3b - 5$ from the sum of $7a + 3b + 9$ and $6a + 5b + 1$

(iv) What must be added to $3x^3 + 5x - x^2 - 5$ to get $2x^2 - 2x - 2x^3 + 2$?

(v) What must be subtracted from $6p^3 + 6p - p^2 - 7$ to get $-6p^2 + 8 - 2p$?

Long answer type questions

10. Find the product

(i) $3x(x^2 - 2x)$ (ii) $2xy(x^2 - y^2)$

(iii) $-3xy(x^2y - 2xy^2)$ (iv) $\frac{xy}{5}(-2x - 5y)$

(v) $(4abc + 3ab) \times (-4b^2c)$ (vi) $(2a - 6b)(a + 4b)$

(vii) $(2x - 3y)(4x^2 + 6xy + 9y^2)$ (viii) $(a - 3b + 5)(2a + 2b - 6)$

(ix) $\left(\frac{3x}{7} + \frac{2y}{3}\right)\left(\frac{3x}{7} - \frac{2y}{3}\right)$ (x) $\left(\frac{x}{2} - \frac{3y}{5}\right)(2x + 5y)$

11. Divide the following

(i) $16x^3$ by $4x$ (ii) $26x^2y$ by $2xy$

(iii) $4.8a^4b^4c^4$ by $24a^2b^2c^2$ (iv) $-12x^2y^4$ by $-4xy^3$

(v) $-20p^3q^3r^3$ by $-10p^2q^2r^2$ (vi) $(-40 + 80a) \div 8a$

12. Draw a tree diagram for each of the expressions

(i) $2a + 4b$ (ii) $-7a^2 + 15ab$ (iii) $4x^2y^3 + 6x$

High order thinking skills (HOTS)

13. If $2x - 2 = 4$ and $0.05y = 0.15$, find the value of $y^2 - x^2$.

14. Find the area of a rectangle whose length and breadth is $(x + 2y)$cm and $(3x - 2y)$ cm respectively.

15. Fill in the blanks

(i) xyz is an algebraic expression which is a .... (monomial, binomial, trinomial)

(ii) Two like terms differ only in their .... coefficients.

(iii) The coefficient of P in $-\frac{2}{3}Px^2$ is ....

(iv) The sum of $-11x$ and $7x$ is ....

(v) The value of $-7(a + b + c) - \{-5\,(a + b + c)\}$ is............

(vi) The algebraic expression representing 2 diminished from 3 times x is..................

(vii) The algebraic expressions $-3x^2y$ and y^3 are .... terms.

(viii) While subtracting two like terms, their numerical coefficients are ....

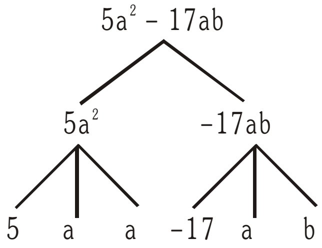
(ix) If |||| + ≡ + □ represents $4x^2 + 3y^2 + z$, then the algebraic expression for ≡ + □□□ + |. ....

(x) $6ab^3 + ... + 2b^2c + 8abc - 14b^3a \quad = 15b^2c + ... + 20abc - 12abc$

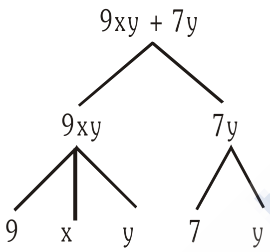
ANSWERS

CHECK POST-1

1. (i)



(ii)



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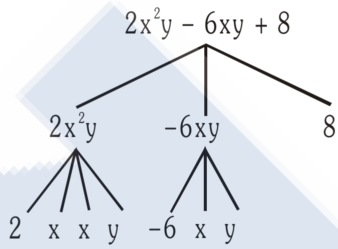


2. $2x^2 + 3xy - y^2$ **3.** $6a + 5b - 2$ **4. (i)** $p = -6$, **(ii)** $p = 2$

5. 8

EXCERCISE-1

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	D	A	C	B	B	B	B	C	C	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	B	B	A	C	B	C	A	A	A	B
Que.	**21**	**22**	**23**	**24**	**25**					
Ans.	C	C	C	C	C					

EXCERCISE-2

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	C	D	A	D	D	C	C	D	D
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	B	D	B	C	A					

EXCERCISE-3

1. (i) $(6a, -7a), (5b, -3b)$ **(ii)** $(11x, 8x, -6x, 5x), (-3y, 6y, y)$

(iii) $(6m, -10m), (8mn, -16mn, 14mn), (-8m^2, 15m^2)$ **(iv)** $(14x^2y, 5yx^2), (6xy, -6yx), (2xy^2, 14y^2x)$

2. (i) Binomial, 1 **(ii)** Binomial, 2 **(iii)** Binomial, 8

(iv) Multinomial, 4 **(v)** Multinomial, 8 **(vi)** Multinomial, 9

3. (i) $3q$ **(ii)** $2q - p$ **(iii)** $3x - 3y + 3z$

(iv) $25p^2 + 2pq - 15q^2$

4. (i) $2a$ **(ii)** $10 + 12x$ **(iii)** $12 + 4bc$

5. (i) $-10, x^2y$ **(ii)** $\frac{6}{13}, a^2bc$ **(iii)** $\frac{-5}{9}$, abc **(iv)** -1, pq

(v) $\frac{-5}{9}, \frac{xy}{z}$ **(vi)** $-8, x^2y^2z^2$

6. **(i)** and **(iii)** are polynomials

7. **(i)** $4a^2 + a + 13$ **(ii)** $q^2 + 2pq + 12r^2 + 12pr$ **(iii)** $6a^2b - 5ab^2 - 4abc - 3b^2c$

(iv) $\frac{-x}{15} - \frac{11y}{12}$ **(v)** $\frac{15}{2}x^2 - \frac{19}{6}x + 3$ **(vi)** $\left(x^3 + \frac{7x^2}{2} - 3\right)$

8. **(i)** $4p^2 - 10p^2q + 8pq^2 + q^2$ **(ii)** $3p^3 - 4p^2 - 6p - 8$

(iii) $2a^4 - 4a^2b^2 + 8a^3b - 8a^2b - 3b^4$ **(iv)** $\frac{22}{15}x^2 - \frac{5}{6}x - 2$ **(v)** $-x^2 - \frac{1}{12}y^2$

9. **(i)** $3a^2 + 7b + 11$ **(ii)** $18p + 11q + 20$ **(iii)** $2a + 8b + 8$

(iv) $-5x^3 - 7x + 3x^2 + 7$ **(v)** $6p^3 + 8p + 5p^2 - 15$

10. **(i)** $3x^3 - 6x^2$ **(ii)** $2x^3y - 2xy^3$ **(iii)** $-3x^3y^2 + 6x^2y^3$ **(iv)** $-\frac{2}{5}x^2y - xy^2$

(v) $-16ab^3c^2 - 12ab^3c$ **(vi)** $2a^2 + 2ab - 24b^2$ **(vii)** $8x^3 - 27y^3$

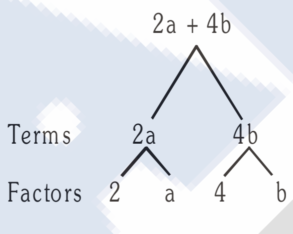
(viii) $2a^2 - 6b^2 - 4ab + 4a + 28b - 30$ **(ix)** $\frac{9}{49}x^2 - \frac{4}{9}y^2$

(x) $x^2 - 3y^2 + \frac{13}{10}xy$

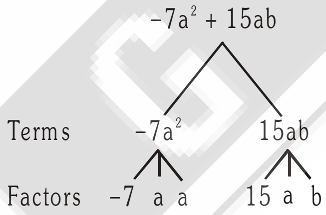
11. **(i)** $4x^2$ **(ii)** $13x$ **(iii)** $0.2a^2b^2c^2$ **(iv)** $3xy$

(v) $2pqr$ **(vi)** $10 - \frac{5}{a}$

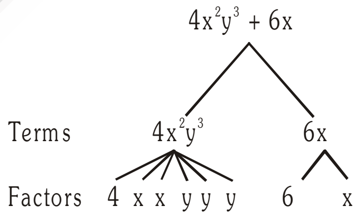
12. **(i)**



(ii)



(iii)



13. 0 **14.** $(3x^2 + 4xy - 4y^2)cm^2$

15. **(i)** Monomial **(ii)** numerical **(iii)** $-\frac{2}{3}x^2$ **(iv)** $-4x$

(v) $-2(a + b + c)$ **(vi)** $3x - 2$ **(vii)** unlike **(viii)** subtracted

(ix) $4y^2 + 3z + x^2$ **(x)** $13b^2c, -8ab^3$

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